**EXPERIMENT 1: Characteristics of Passive and Active Filters**

**Objective:** To understand the behavior of passive filters and active filters based on RC networks and operational amplifier

In circuit theory, a filter is an electrical network that alters the amplitude and / or phase characteristics of a signal with respect to frequency. Ideally, a filter will not add new frequencies to the input signal, nor will it change the component frequencies of that signal, but it will change the relative amplitudes of the various frequency components and / or their phase relationships. Note that the change in phase is directly related to the change in time (Think Delay).

The voltage transfer function of a filter is defined by $H(S) = \frac{V_{OUT}(S)}{V_{IN}(S)}$.

The transfer function for an order network (one with 'n' capacitors and inductors) is defined by

$$H(S) = H_0 \frac{S^n + b_{n-1}S^{n-1} + b_{n-2}S^{n-2} + \ldots + b_1S + b_0}{S^n + a_{n-1}S^{n-1} + a_{n-2}S^{n-2} + \ldots + a_1S + a_0}$$

The filter characteristics is completely defined by the filter coefficients $a_i$ and $b_i$.

**Butterworth Characteristics**

It exhibits nearly flat pass band with no ripple. The roll-off is smooth and monotonic, with a low-pass or high-pass rolloff rate of 20dB/decade (6dB/Octave) per pole. Note that the phase response of Butterworth filter is non linear.
Butterworth Quadratic Factors (up to n=6) are given in the table 1.1 [Normalized]

<table>
<thead>
<tr>
<th>n</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(S+1)</td>
</tr>
<tr>
<td>2.</td>
<td>(S^2+1.4142S+1)</td>
</tr>
<tr>
<td>3.</td>
<td>(S+1)(S^2+S+1)</td>
</tr>
<tr>
<td>4.</td>
<td>(S^2+0.7654S+1)(S^2+1.8478S+1)</td>
</tr>
<tr>
<td>5.</td>
<td>(S+1)(S^2+0.618S+1)(S^2+1.6180S+1)</td>
</tr>
</tbody>
</table>

Table 1.1
This shows, fifth order Butterworth filter can be realized by cascading one first order and two second order sections.

Gains for Butterworth Filter

<table>
<thead>
<tr>
<th>Poles</th>
<th>Roll-off (Decade)</th>
<th>1st Section (1 or 2 Pole)</th>
<th>2nd Section (2 Poles)</th>
<th>3rd Section (2 Poles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>20dB</td>
<td>Optional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>40dB</td>
<td></td>
<td>1.586</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>60dB</td>
<td>Optional</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>80dB</td>
<td></td>
<td>1.152</td>
<td>2.235</td>
</tr>
<tr>
<td>5.</td>
<td>100dB</td>
<td>Optional</td>
<td>1.382</td>
<td>2.382</td>
</tr>
<tr>
<td>6.</td>
<td>120dB</td>
<td></td>
<td>1.068</td>
<td>1.586</td>
</tr>
</tbody>
</table>

Table 1.2

Passive Filters and Active Filters

Passive filters include the elements such as R, L or C. Active filters employ transistors or op-amps in addition to R, L or C. RC filters are commonly used for audio applications, and LC filters are employed at RF or high frequencies.

Active filters can provide gain, and the inter-stage loading problem is reduced in case of active filters.

A. Passive Filters

1. Determine the cut-off frequency of the RC low pass filter. Given R=1K and C=1uF.

\[ f_c = \frac{1}{2\pi RC} \]
2. Using the phase of Bode plotter find the phase shift at the cut-off frequency and find the maximum phase lag that one RC section can offer.
3. Verify that the filter rolloff rate is -20dB/decade.
4. To realize a HP responses simply interchange the role of R and C.
5. To realize a BP response simply cascade a HP and LP filter. The critical frequency is determined by the geometric mean of critical frequencies of a LP and HP filter respectively.
6. Verify the time domain analysis using oscilloscope.
7. Connect another RC section to make second order LPF and find the BW. The BW is given by \( \frac{0.374}{CR} \).
8. Verify that the roll off rate is 40dB/decade above the cut-off.
9. Find the maximum phase lag that two RC sections can offer

Note: You can use AC analysis also instead of Bode Plotter.

B. Rise Time and Bandwidth
Verify the relation between the rise time and bandwidth using low pass filter.

\[
TR = 0.35 \frac{BW}{z}
\]

1. Set the function generator to square wave @ 80Hz and 10V.
2. Use Measure in the digital oscilloscope to measure the rise time.

If you use an oscilloscope with inherent rise time of \( t_r \) and if you are measuring a signal with high speed rise time then use a formula,

\[
t_o = \sqrt{t_m^2 - t_r^2}
\]

Where, \( t_m \) is the measured rise time and \( t_r \) is the oscilloscope inherent rise time to find the actual rise time of signal with no measurement error.
C. Design a first order LP active filter having a cut-off frequency at 1 kHz with an optional pass band gain. Verify that the roll-off rate is 20dB/decade above the cut-off.

We assume the pass band gain of 6dB i.e. 2. The circuit is shown below.

The cut-off frequency is given by $f_c = \frac{1}{2\pi RC}$. Here, $R = 15.9K$ and $C = 0.01\mu F$.

i) To obtain a high frequency responses simply change the role of $R$ and $C$.

ii) Obtain the magnitude response for a cut-off frequency of 2 kHz with pass band gain of 6dB.

**A Word Of Caution** The Response of HP active filter (op-amp based) will be seen like band pass response. WHY?? [I am Slew rate ⊗. I am GBWP ⊗]
D. **Design a second order Sallen key LPF with upper cut-off frequency of 7.96 kHz. Optimize the filter for Butterworth response.**

![Sallen Key LPF Circuit](image)

The feedback capacitor ($C_A$) provides the feedback for shaping the response near the edge of the pass band.

**Design Equations**

$$f_c = \frac{1}{2\pi \sqrt{R_A R_B C_A C_B}}$$

Gain = 1.586 = \left(1 + \frac{R_1}{R_2}\right)

where, $R_1$ is the feedback resistor. The gain is found from Table 1.2.

To design a second order Sallen Key high pass filter simply interchange the role or resistors and capacitors. The design equations are same. To design a second order Sallen Key BP filter simply cascade second order HP and second order LP Sallen Key filter. The centre frequency will be the geometric mean of low pass and high pass critical frequencies i.e.

$$f_o = \sqrt{f_{c1} f_{c2}}$$

E. **Design a narrow band reject filter of 60Hz.**

We use Twin-T notch filter. The notch frequency is given by

$$f_N = \frac{1}{2\pi RC}$$
F. **Design of an All Pass Filter**

All pass filter passes all the frequency components of the input signal without attenuation, while providing predictable phase shifts for different frequencies of the input signal. When a signal is transmitted through a communication channel (*REMEMBER COMMUNICATION CHANNEL IS A LOW PASS FILTER*), they undergo change in the phase. To compensate for this change in phase all-pass filter is required. All pass filters are also called phase correctors or delay equalizers.

In summary, all pass filters can provide phase change so they are used as a phase correctors.

**Circuit Diagram**

This is nothing but like a first order low pass filter (*Which you have seen in question no. C*), the only difference is that the grounded 10K resistor is connected to the input.

**Design Equation**

\[
\phi = -2\arctan(2\pi fRC)
\]

Note that the phase shift is the function of input frequency. The phase shift is in degree.
Design a phase corrector that provide -90 degree phase shift for a input signal of 1kHz frequency.

Given is \( f = 1 \) kHz. Select a value of R or C and find the value of C or R using the design equation for \( \phi = -90^\circ \). The phase response and magnitude response is shown below.

Design a phase corrector that provide +90 degree phase shift for a input signal of 1kHz frequency.

For this simple interchange the position of R and C.

Note the upper frequency limitation by op-amp and note the gain of the filter.