Course: MCSC 202  ASSIGNMENT - III(Finite differences and Interpolation)

Problem 1  Prove the following relations where the operators have their usual meanings. 
(i) $\delta^2 E \equiv \Delta^2$ 
(ii) $E^{-1/2} \equiv \mu - \delta/2$  
(iii) $1 + \delta^2/2 \equiv (1 + \delta/2)^2$  
(iv) $\Delta^2 x^m = m(m-1)x^{m-2}$ if $m \in \mathbb{N}$  
(v) $u_0 - u_1 + u_2 - \cdots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \cdots$  
(vi) $u_1 x + u_2 x^2 + u_3 x^3 + \cdots = \frac{x}{1+x} u_1 + \frac{x^2}{(1-x)^2} \Delta u_1 + \frac{x^3}{(1-x)^3} \Delta^2 u_1 + \cdots$

Problem 2  Derive Newton’s backward difference interpolation formula. The population of a country in the decennial census were as under. Estimate the population for the year 1895.

<table>
<thead>
<tr>
<th>Year, $x$</th>
<th>1891</th>
<th>1901</th>
<th>1911</th>
<th>1921</th>
<th>1931</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population, $y(\times 10^4)$</td>
<td>46</td>
<td>66</td>
<td>81</td>
<td>93</td>
<td>101</td>
</tr>
</tbody>
</table>

Problem 3  In an examination the number of candidates who obtained marks between certain limits were as follows:

<table>
<thead>
<tr>
<th>Marks</th>
<th>0-19</th>
<th>20-39</th>
<th>40-59</th>
<th>60-79</th>
<th>80-99</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of candidates</td>
<td>41</td>
<td>62</td>
<td>65</td>
<td>50</td>
<td>17</td>
</tr>
</tbody>
</table>

Estimate the number of candidates who obtained fewer than 70 marks.

Problem 4  Use Stirling formula and Bessel formula to find $u_{32}$ from the following table: $u_{20} = 14.035, \quad u_{25} = 13.674, \quad u_{30} = 13.257, \quad u_{35} = 12.734, \quad u_{40} = 12.089, \quad u_{45} = 11.309$

Problem 5  Given the table of values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>150</th>
<th>152</th>
<th>154</th>
<th>156</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \sqrt{x}$</td>
<td>12.247</td>
<td>12.329</td>
<td>12.410</td>
<td>12.490</td>
</tr>
</tbody>
</table>

evaluate $\sqrt{155}$ using Lagrange’s interpolation formula.

Problem 6  Deduce Newton’s forward and Newton’s backward difference interpolation formula as a particular case of Newton’s divided difference interpolation formula. If $f(x) = 1/x$, prove that $[x_0, x_1, \cdots, x_r] = \frac{(-1)^r}{x_0 x_1 \cdots x_r}$.

Problem 7  Locate and correct the error in the following table of values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
<th>5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4.32</td>
<td>4.83</td>
<td>5.27</td>
<td>5.47</td>
<td>6.26</td>
<td>6.79</td>
<td>7.23</td>
</tr>
</tbody>
</table>

Problem 8  Obtain a value of $x$ when $f(x) = 19$, given the following values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

Problem 9  Using Lagrange’s interpolation formula find a polynomial which passes through the points $(0, -12), (1, 0), (3, 6), (4, 12)$.

Problem 10  By means of Newton’s divided difference formula find the polynomial of the lowest degree which assumes the values $3, 12, 15, -21$, when $x$ has the values $3, 2, 1, -1$ respectively.

Problem 11  Explain the term interpolation and extrapolation. Derive Lagrange’s interpolation formula. On what assumptions it is derived and what are the situations most suited for its application?

Problem 12  The following table of values represents a polynomial of degree $n \leq 3$. Locate any error in the table of values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2.00</td>
<td>2.11</td>
<td>2.28</td>
<td>2.39</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Problem 13  Derive Gauss forward and backward central difference interpolation formulas. Using Gauss backward formula, find the value of $y_{15}$ given that $y_{10} = 111.803399, y_{15} = 111.848111, y_{20} = 111.892806, y_{25} = 111.937483$.

Problem 14  Find the cubic polynomial which takes the following values: $y(1) = 24, y(3) = 120, y(5) = 336$, and $y(7) = 720$. Hence, or otherwise, obtain the value of $y(8)$. 

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