KATHMANDU UNIVERSITY
Additional Exercise Sheet

Level: CS/CM II\textsuperscript{nd} Year/ II\textsuperscript{nd} Sem
Course: MCSC-202 (Numerical Methods)
Instructor: Dr. Samir Shrestha

Numerical Solutions of ODE (IVP)

Task 1: Solution by Taylor’s and Picard’s

(i) Given that \( \frac{dy}{dx} - 1 = xy \) with \( y(0) = 1 \), obtain the Taylor series for \( y(x) \) and compute \( y(0.1) \) and \( y(0.2) \).

(ii) Using Picard’s method, obtain the solution of \( y'(x) = xe^x \) with \( y(0)=0 \). Tabulate the values of \( y(0.1), y(0.2) \) and \( y(0.3) \).

Task 2: Euler’s and Modified Euler’s Methods

(i) Using Euler’s method, solve \( \frac{dy}{dx} + 2y = 0 \), \( y(0)=1 \) by taking step size \( h=0.1 \) to compute \( y(0.1), y(0.2) \) and \( y(0.3) \).

(ii) Given \( \frac{dy}{dx} = x^2 + y \), \( y(0)=1 \) determine \( y(0.02), y(0.04) \) and \( y(0.06) \) using Euler’s modified method.

Task 3: Runge-Kutta Methods

(i) Use Runge-Kutta second order method to solve \( 10 \frac{dy}{dx} = x^2 + y^2 \), \( y(0)=1 \) for the interval \( 0 < x \leq 0.4 \) with \( h=0.1 \).

(ii) Using fourth order RK method to estimate \( y(0.5) \) of the equation \( \frac{dy}{dx} = \frac{x}{y} \), \( y(0)=1 \) with \( h = 0.25 \).

System of Linear Equations

Task 4: Consistent and Inconsistent

Determine following systems of linear equations have unique, infinitely many or no solution(s) by finding the ranks of coefficient and augmented matrices:

(i) \(-2x + y + 3z = 12\) \hspace{1cm} (ii) \(2x + 2y + 4z = 18\) \hspace{1cm} (iii) \(x + y + z = 20\)

\(x + 2y + 5z = 4\) \hspace{2cm} \(x + 3y + 2z = 13\) \hspace{2cm} \(2x - 3y + z = -5\)

\(6x - 3y - 9z = 24\) \hspace{2cm} \(3x + y + 3z = 14\) \hspace{2cm} \(3x - 2y + 2z = 15\)

Hint: Use MATLAB command \texttt{rank} to find \texttt{rank} of the matrices.
Task 5: LU Decomposition Method

Solve the system $2x + 3y + 2z = 14$ by using LU decomposition method.

$$x + 2y + 3z = 14$$

Task 6: Iterative Methods

(i) Solve the system $10x_1 + x_2 + x_3 = 12$

(ii) Solve the system $2x_1 + 10x_2 + x_3 = 13$

Solve the system $2x_1 + 2x_2 + 10x_3 = 14$

Solve the system $13x_1 + 5x_2 - 3x_3 + x_4 = 18$

Solve the system $2x_1 + 12x_2 + x_3 - 4x_4 = 13$

Solve the system $3x_1 - 4x_2 + 10x_3 + x_4 = 29$

Solve the system $2x_1 + x_2 - 3x_3 + 9x_4 = 31$

Gauss-Seidel iterative methods with initial approximation $(0, 0, 0, 0)$. 